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error of a R.A. in the Cape Catalogue = $\pm 0^s{\cdot}027$ cosec N.P.D., and the probable error of a N.P.D. = $\pm 0''{\cdot}35$. Combining these values with the probable errors of a single $\Delta\alpha$ sin N.P.D. and a single $\Delta\delta$ given above, it appears that the probable errors of a R.A. and N.P.D. of the Argentine General Catalogue are $\pm 0^s{\cdot}038$ cosec N.P.D. and $\pm 0''{\cdot}42$ respectively.

On the Reduction of Star Places by Bohnenberger's Method.

By Professor Truman Henry Safford.

Every astronomer who has had much to do with the reduction of star places from one epoch to another is aware that in case the star be very near the pole, or the epochs very distant, the method invented by Bohnenberger must replace the ordinary employment of annual precessions and secular variations.

It is easy enough to see that when the product of the interval by the tangent of the star's declination reaches a certain amount the precession series is no longer convergent. If I can rely upon my remembrance of a mathematical investigation found in many text-books, the series

$$1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\tfrac{1}{7}-\tfrac{1}{11}, \text{ etc. for } \tfrac{1}{4}\pi = {\cdot}78539\ldots$$

is just on the line between convergence and non-convergence; and when the series, by which the right ascension of a star is expressed in terms of the time, contains as a part of itself a series of this degree of convergence it is no longer practicable to use it.

From Bessel's investigation on pages x and xi of the introduction to the *Tabulæ Regiomontanæ*, one can readily enough see that a star's right ascension as a function of the time contains terms of the form

$$\begin{aligned} & tn \tan\delta \sin\alpha \\ & + \frac{t^2n^2}{2}\tan^2\delta \sin 2\alpha \\ & + \frac{t^3n^3}{3}\tan^3\delta \sin 3\alpha \\ & + \frac{t^4n^4}{4}\tan^4\delta \sin 4\alpha. \\ & + \text{\&c.} \end{aligned}$$

together with other terms multiplied by the same powers of tan δ, but lower power of the time. The terms here given are those which determine the convergence when tan δ is very great.

If, now, $tn \tan\delta = 1$ we shall have a series which exhibits precisely the same kind of lack of convergence as the well-known numerical arc which I quoted before.

As n is roughly $20''$ or $\frac{1}{10000}$ of radius, it is plain that this

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absolute failure of convergence takes place when $t\tan\delta$ is about 10,000; or for a star 1° from the pole is about 180 years.

Using similar considerations I have been in the habit of selecting the epochs for the use of Bohnenberger's formula as follows.

We have tables for the secular variation, and for the term multiplied by the third power of the time. While these tables are not quite complete enough for close polar stars they are a very great help in checking calculations, and it is easy enough to use them or to calculate the terms depending upon the third, and by their variations from year to year those depending upon the fourth power of the time. So that when the term

$$\frac{t^5 n^5}{5}\tan^5\delta\sin 5\alpha$$

is insensible, it is not necessarry to employ the trigonometrical formula.

But it will be safer not to consider the sine of five times the right ascension as a factor; as, when it is casually small, the next factor sin 6α may be large; and we may require

$$\frac{(tn\tan\delta)^5}{5}$$

as the quantity to be made $0''\cdot01$ of arc. We shall then have

$$\frac{tn\tan\delta}{206264''\cdot8}=\sqrt[5]{\frac{0''\cdot05}{206264''\cdot8}},$$

or

$$5\log\frac{tn\tan\delta}{206264''\cdot8}=3\cdot38455-10,$$

or

$$\log\frac{tn\tan\delta}{206264''\cdot8}=8\cdot67691-10,$$

or

$$\log tn\tan\delta=3\cdot99134.$$

Taking $\log n$ for 1900, according to Struve $=1\cdot30216$, we shall have

$$\log t\tan\delta=2\cdot68918,$$

or

$$t\tan\delta=488\cdot9 \text{ years}.$$

For *Polaris* in 1887, whose declination is 88° 42′·5, the interval here indicated is about 11 years. That is to say, the right ascension of *Polaris* can now be carried forward or back 11 years by the use of the first four differential coefficients of the right ascension with respect to the time; so that a trigonometrical computation for every 20 years is amply sufficient to afford a check at the beginning of one period and the end of the next preceding.

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For a star in the declination 88° 58′ the single interval t is rather less than 9 years, and the double rather more than 17; so that to compute a series of places from year to year of λ *Ursæ Minoris* or Groombridge 1119 = B.A.C. 2320 (*Connaissances des Temps*) the trigonometrical computation should be employed every 16 years at least, to make sure of 0″·01 in the right ascension. Similar consideration should be applied to find out when other terms become insensible, or less than 0″·01.

Suppose we employ the general formula

$$\frac{1}{p}\left(tn \tan \delta\right)^{p} = 0''\cdot 01,$$

making p successively 3, 4, 5 . . . and we shall have

$$t \tan \delta = \left(\frac{p \times 0''\cdot 01}{206264''\cdot 8}\right)^{\frac{1}{p}} \frac{206264''\cdot 8}{n},$$

as given in the following little table:—

p	$t \tan \delta$ years.	t for *Polaris.* y	t for λ *Ursæ Minoris.* y
3	54·1	1·2	1·0
4	215·9	4·9	3·9
5	488·9	11·	8·9
6	837·3	19·	15·
7	1225·	28·	22·
8	1625·	37·	30·
9	2021·	46·	37·
10	2403·	54·	44·

The formula is

$$\log t \tan \delta = 4\cdot 01226 - \frac{7\cdot 31443}{p} + \frac{\log p}{p}.$$

Practically, anyone who attempts to use the higher terms up to the 10th in reducing star places will find it a very uncertain and perplexing matter, far less easy than the judicious employment of the trigonometrical formulæ.

If in any given case, like that of λ *Ursæ Minoris* for example, it is desired to extend the series to the 5th term, this can be practically accomplished as follows:—

The highest part only in power of $\tan \delta$ of the 4th term will be

$$t^4 \frac{n'^4}{4} \tan^4 \delta \sin 4\alpha,$$

and that of the 5th term

$$t^5 \frac{n'^5}{5} \tan^5 \delta \sin 5\alpha,$$

which last will doubtless be sufficiently approximate to the total

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value of the term for 10 years, as it cannot exceed 0″·02 for this period, which is at any rate quite insensible when reduced to the parallel of the star.

If, indeed, we choose to take 0″·01 *after reduction to the parallel* as our limit of accuracy in computation, we may practically employ the t corresponding to a term farther along in the series than we intend to employ; but a skilled computer will soon find the practical limits beyond which these theoretical arcs may be extended without damage.

In this way it will be seen that for stars not very near the pole the terms beyond the ordinary "secular variation" soon become sensible. As a long and valuable memoir by two Austrian astronomers, Herz and Strobl, which I have seen but do not yet possess, relates, in part, to this subject in its application to Auwers's fundamental catalogue, I will only say here that for $\delta = 45°$, t should not be more than 54 years to secure (without examination of the special case) the insensibility to less than 0″·01 of the error caused by omitting the third term. Professor Argelander in fact calculated these values for all declinations over 39°; his greatest interval was 120 years (1755–1875), which agrees well enough with my table.

The actual calculation by Bohnenberger's method requires the use of three quantities $z+\lambda$, $z'-\lambda'$, and θ, which depend partly upon the epoch from which, and that to which, the reduction is effected; and partly upon the epoch taken as the fundamental one for the precession, constants, and so forth.

In the use of Struve's precessions, now entirely general, the fundamental epoch is of course 1800. I have used 1855 as the general date from and to which the places are to be reduced, following Carrington for reasons of convenience. The same date had been also adopted by Dr. Gould, and was afterwards used by Professor Boss; and I have not thought it worth while to change at present to a later epoch. Carrington's catalogue affords (for all close northern polars) approximate positions at once for this year; it also lies between the two Radcliffe catalogues, which can by their own data be readily reduced to it; and is nearly identical with the general mean date (by weight) of the existing catalogues of these northern stars. The method which I employ makes this a convenience, and at the same time renders it needless to select a later date for the constants $z+\lambda$, &c.

If a change is to be made in this respect I think the next year adopted should be 1900, so far as these quantities are concerned. As a matter of curiosity I desired to follow the changes in the place of *Polaris*, and so computed some values of their constants up to 2100. In the table which follows I give them; those to be added to the right ascension are expressed in time. This is now rendered a matter of the greatest convenience by the admirable tables of the previously mentioned astronomer

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Norbert Herz,[*] published by Teubner at Leipzig in 1885; a book which I find indispensable in all computations of precession.

At a future time I intend to publish the results of my discussion of the relative movements of *Polaris* and the north pole; it is enough to say here that the nearest approach takes place, according to my calculation, some time in the year 2102 at a north declination of 89° 32′ 23″; the star will reach 89° about 1944, and be for about 300 years within a degree of the pole.

The tables have been pretty thoroughly checked in various ways. They were computed for every twenty years with seven-figure logarithms (Schrön's), using the additional marks to indicate the half of the seventh decimal, and carrying the interpolations to the eighth figure; and were compared with values given by Dr. Gould and Professor Boss for various dates, and by differences for intermediate years.

Moreover, the position of *Polaris* was brought up to 1900 from 1855, and then to 2000 and 2100 by both tables, with very slightly different results; in neither co-ordinate more than 0″·002 in arc of a great circle. I had no ten-figure table at hand, but it hardly seemed necessary.

The simplest method of discussing a long series of polar-star places collected from the various catalogues is to assume a position for 1855·0 from a careful preliminary discussion; reduce it to the various epochs of Table I. for which there are observations, and compare with observations. The benefit of this process is that defective or erroneous right ascensions do not affect the reductions of the declination; while the error in the assumed place is readily found and allowance made if necessary.

TABLE I.—*For Bohnenberger's Method.*

(The reductions are *from* 1855·0 to the tabular dates.)

Date.	$z+\lambda$	$z'+\lambda'$	θ
	m s	m s	′ ″
1755	−2 33·588	−2 33·506	−33 25·987
1760	2 25·909	2 25·835	31 45·671
1780	1 55·196	1 55·142	25 4·423
1790	1 39·841	1 39·791	21 43·816
1800	1 24·486	1 24·438	18 23·202
1810	1 9·133	1 9·083	15 2·603
1815	1 1·456	1 1·404	13 22·305
1820	0 53·780	0 53·724	11 42·010
1828	0 41·497	0 41·436	9 1·542
1830	0 38·427	0 38·364	8 21·426
1835	0 30·751	0 30·683	6 41·137
1840	0 23·075	0 23·001	5 0·849

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[*] *Siebenstellige Logarithmen der trigonometrischen Functionen für jede Zeitsecunde*, 186 pp. 8vo.

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Date.	$z+\lambda$	$z'+\lambda'$	θ
	m s	m s	′ ″
1842	0 20·005	0 19·928	4 20·735
1845	0 15·400	0 15·318	3 20·564
1850	0 7·724	0 7·635	1 40·281
1852	0 4·654	−0 4·562	−1 0·168
1855	−0 0·049	+0 0·049	0·000
1860	+0 7·627	0 7·733	+1 40·279
1864	0 13·766	0 13·881	3 0·500
1865	0 15·301	0 15·418	3 20·555
1870	0 22·976	0 23·104	5 0·830
1872	0 26·046	0 26·178	5 40·939
1875	0 30·651	0 30·790	6 41·102
1880	0 38·325	0 38·477	8 21·372
1885	0 46·000	0 46·165	10 1·640
1890	0 53·674	0 53·853	11 41·905
1900	1 9·022	1 9·232	15 2·428
1920	1 39·715	1 39·997	21 43·445
1940	2 10·407	2 10·772	28 24·422
1960	2 41·096	2 41·558	35 5·355
1980	3 11·782	3 12·354	41 46·244
2000	3 42·467	3 43·161	48 27·085
2020	4 13·149	4 13·978	55 7·879
2040	4 43·829	4 44·807	1° 1 48·622
2060	5 14·507	5 15·646	1 8 29·312
2080	5 45·182	5 46·496	1 15 9·947
2100	6 15·856	6 17·357	1 21 50·525

TABLE II.—*For Bohnenberger's Method.*

(Reduction from 1900 to tabular date.)

Date.	$z+\lambda$	$z'+\lambda'$	θ
	m s	m s	′ ″
1920	+0 30·532	+0 30·926	+ 6 41·025
1940	1 1·223	1 1·701	13 22·013
1960	1 31·912	1 32·486	20 2·962
1980	2 2·598	2 3·282	26 43·872
2000	2 33·281	2 34·088	33 24·739
2020	3 3·962	3 4·905	40 5·562
2040	3 34·641	3 35·732	47 46·339
2060	4 5·318	4 6·570	53 27·068
2080	4 35·992	4 37·419	1° 0 7·746
2100	5 6·665	5 8·278	1 6 48·373

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The formulæ for reductions from 1855 or 1900 to any tabular date are (here $æ$, δ are the position for the fundamental epoch, $æ'$, δ' those for the tabular date).

$$\begin{aligned}
\mathrm{A} &= æ + z + \lambda \\
m \cos \mathrm{M} &= \cos \delta \cos \mathrm{A} \\
m \sin \mathrm{M} &= \sin \delta \\
\cos \delta' \sin \mathrm{A}' &= \cos \delta \sin \mathrm{A} \\
\cos \delta' \cos \mathrm{A}' &= m \cos (\mathrm{M} + \theta) \\
\sin \delta' &= m \sin (\mathrm{M} + \theta) \\
æ' &= \mathrm{A}' + z' - \lambda'.
\end{aligned}$$

Other formulæ giving the same results are frequently employed, but I find these on the whole the most convenient.

On the Orbit of p Eridani. By J. E. Gore.

In previous investigations on the orbit of this southern binary star it seems to have been assumed that Dunlop's measure in 1825 is probably erroneous, owing to the difficulty experienced in making it fit satisfactorily into an orbit with the other measures. Assuming, however, the approximate correctness of Dunlop's measure, and using some recent measures by Mr. Tebbutt (N.S.W.), and by Mr. Pollock at the Sydney Observatory, I have computed the following provisional elements:—

Elements of p Eridani.

$P = 302{\cdot}37$ years.
$☊ = 135^\circ\ 0'$ (1870·0).
$T = 1823{\cdot}55$.
$\lambda = 240^\circ\ 0'$.
$e = 0{\cdot}674$.
$a = 6''{\cdot}96$.
$\gamma = 38^\circ\ 31'$.
$\mu = -1^\circ{\cdot}19$.

The following is a comparison between the recorded measures and the positions computed from the above elements. The computed position-angles have been corrected for the effect of precession to 1870·0.

Epoch.	Observer.	θ_o	θ_c	$\theta_o - \theta_c$	ρ_o	ρ_c	$\rho_o - \rho_c$
		°	°	°	″	″	″
1825·96	Dunlop	343·1	348·6	−5·5	2·5 (est)	2·12	+0·38 ±
1835·00	Sir J. Herschel	302·3	303·9	−1·6	3·65	3·17	+0·48
1845·88	Jacob	276·0	276·7	−0·7	4·16	4·13	+0·03
1846·35	,,	276·5	275·8	+0·7	4·32	4·18	+0·14
1849·82	,,	270·0	269·5	+0·5	...	...	...
1850·80	,,	268·7	267·9	+0·8	...	4·51	...
1851·79	,,	266·4	266·3	+0·1	4·30	4·59	−0·29